MATH 220 HANDOUT 4 - INDUCTION WARMUP

1. We want to prove, by induction, that, for every positive integer n,

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

a) What is the open statement "$P(n)$"?

$P(n) =$

b) What is the statement "$P(1)$"? Why is $P(1)$ true?

$P(1) =$

c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that $P(n-1) \Rightarrow P(n)$).

Assume that

We want to show that

(Inductive step)

2. Let a_n be a sequence such that $a_1 = 1$ and $a_n = na_{n-1}$. We want to prove, by induction, that, for every positive integer n,

$$a_n = n! = n(n-1)(n-2)\cdots 2 \cdot 1.$$

a) What is the open statement "$P(n)$"?

$P(n) =$

b) What is the statement "$P(1)$"? Why is $P(1)$ true?

$P(1) =$

c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that $P(n-1) \Rightarrow P(n)$).

Assume that

We want to show that

(Inductive step)

3. We want to prove, by induction, that, for every positive integer n,

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

a) What is the open statement "$P(n)$"?

$P(n) =$

b) What is the statement "$P(1)$"? Why is $P(1)$ true?

$P(1) =$

c) What is the inductive step? Write out your assumption, your desired conclusion, and the inductive step (i.e., the proof that $P(n-1) \Rightarrow P(n)$).

Assume that

We want to show that

(Inductive step)

4. Prove, by induction, that $2^{n+1} \geq n^2$ for every integer n. (For this problem, you will have to first check $P(1)$ and $P(2)$.)