

## MATH 220 HANDOUT 7 - MORE PROBLEMS ABOUT SETS; POWER SETS

- (1) Let  $A, B$ , and  $C$  be sets. Draw a Venn diagram demonstrating each of the following and then prove each of the following.
- (a) Prove that  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ .
  - (b) Prove that  $A - (B \cap C) = (A - B) \cup (A - C)$ .
  - (c) Prove that  $(A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$ .
  - (d) Prove that if  $A - B \subseteq C$ , then  $\overline{C} \subseteq \overline{A} \cup B$ .
  - (e) Prove that if  $A \subseteq B$ , then  $A \cup B - A \cap B = A - B$ .
- (2) Disprove each of the following statements by giving examples of sets  $A, B, C$  demonstrating that the statement is false. (I.e., give a counterexample.)
- (a) For all sets  $A, B, C$ ,  $A \cup (B \cap C) = (A \cup B) \cap C$ .
  - (b) For all sets  $A, B, C$ ,  $A \cap (B \cup C) = (A \cap B) \cup C$ .
  - (c) For all sets  $A, B, C$ , if  $A, B$  are subsets of  $C$ , then  $(C - A) - B = C - (A - B)$ .
  - (d) For all sets  $A, B, C$ , if  $A, B$  are subsets of  $C$  and if  $A \subseteq B$ , then  $(C - A) \subseteq (C - B)$ .

**Power Sets**

- (a) Write out 4 elements of
- (i)  $P(\{0, 1, 2, 3, 4\})$ ;
  - (ii)  $P(\mathbf{N})$ ;
  - (iii)  $P(\mathbf{R})$ .
- (b) Let  $A = \{0, 1, 2\}$ . Circle whichever of the following statements are true.
- (i)  $\{0\} \subseteq P(A)$ ;
  - (ii)  $\{1, 2\} \in P(A)$ ;
  - (iii)  $\{\{0, 1\}, \{1\}\} \subseteq P(A)$ ;
  - (iv)  $\emptyset \in P(A)$ ;
  - (v)  $\emptyset \subseteq P(A)$ ;
  - (vi)  $\{\emptyset\} \in P(A)$ .
  - (vii)  $\{\emptyset\} \subseteq P(A)$ ;
  - (viii)  $\{1, \{1\}\} \subseteq P(A)$ .
- (c) Let  $A$  and  $B$  be sets. Prove or disprove:
- (i)  $P(A) \cup P(B) \subseteq P(A \cup B)$ .
  - (ii)  $P(A \cup B) \subseteq P(A) \cup P(B)$ .
  - (iii)  $P(A) \times P(B) \subseteq P(A \times B)$ .
  - (iv)  $A = B$  if and only if  $P(A) = P(B)$ .