MATH 220 HANDOUT 9 - FUNCTIONS AND IMAGES

(1) Give an example of a function with each of the following domain/codomains.
   (a) $\mathbf{Z} \to \mathbf{Z} \times \mathbf{Z}$.
   (b) $\mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$.
   (c) $\mathbf{Z} \to \mathbf{Z}_{>0}$.
   (d) $\mathbf{R} \to \mathbf{Z}$.
   (e) $P(\mathbf{R}) \to P(\mathbf{Z})$.
   (f) $\mathbf{Z} \to P(\mathbf{Z})$.
   (g) $P(\mathbf{Z}) \to \mathbf{Z}$.

(2) Draw a picture of two different functions from $\{1, 2, 3\} \to \{4, 5\}$.

Images

(3) Let $f\colon A \to B$ be a function. Finish the following sentence: an element $b \in B$ is not in the image of f if $\cdots$

(4) Compute the image of the following functions:
   (a) $g\colon \mathbf{Z} \to \mathbf{Z}$, where $g(x) = 2n + 1$;
   (b) $g\colon \mathbf{R} \to \mathbf{R}$, where $g(x) = 2x + 3$;
   (c) $f\colon \mathbf{R} \to \mathbf{R}$, where $f(x) = -x^2 + 1$;
   (d) $\cos\colon \mathbf{R} \to \mathbf{R}$;
   (e) $\tan^{-1}\colon \mathbf{R} \to \mathbf{R}$;

(5) Let $f\colon A \to B$ be a function and let $X, Y \subseteq A$. Prove or disprove each of the following:
   (a) $X \subseteq Y \Rightarrow f(X) \subseteq f(Y)$.
   (b) $X \subseteq Y \Leftarrow f(X) \subseteq f(Y)$.
   (c) $f(X \cup Y) \subseteq f(X) \cup f(Y)$.
   (d) $f(X \cup Y) \supseteq f(X) \cup f(Y)$.
   (e) $f(X \cap Y) \subseteq f(X) \cap f(Y)$.
   (f) $f(X \cap Y) \supseteq f(X) \cap f(Y)$.
   (g) (HW) $f(X) - f(Y) \subseteq f(X - Y)$.
   (h) (HW) $f(X) - f(Y) \subseteq f(X - Y)$.