MATH 220 HANDOUT 11 - INJECTIVITY

(1) Draw a picture of a function which is
   (a) injective

   (b) not injective

(2) Finish the following sentence: a function $f\colon X \to Y$ is *not* injective if ...

(3) Which of the following functions are injective?

(a) **Inj N**: $\mathbf{R} \to \mathbf{R}$; $x \mapsto x^2$.

(b) **Inj N**: $\mathbf{R} \to \mathbf{R}$; $x \mapsto \frac{x+1}{2}$.

(c) **Inj N**: $\mathbf{R} \to \mathbf{R}$; $x \mapsto \cos x$.

(d) **Inj N**: $[0, \pi) \to \mathbf{R}$; $x \mapsto \cos x$.

(e) **Inj N**: $\mathbf{R}_{\geq 0} \to \mathbf{R}$; $x \mapsto \frac{x-1}{x+1}$.

(f) **Inj N**: $\mathbf{R} - \{-1\} \to \mathbf{R}$; $x \mapsto \frac{x-1}{x+1}$.

(g) **Inj N**: $\mathbf{R} \to \mathbf{R}$; $x \mapsto \arctan x$.

(h) **Inj N**: $\mathbf{R}^3 \to \mathbf{R}^2$; $(x, y, z) \mapsto (x, y)$.

(i) **Inj N**: $\mathbf{R}^2 \to \mathbf{R}^3$; $(x, y) \mapsto (x + y, x - y, x^2 + y^2)$.

(j) **Inj N**: $P(\mathbf{R}) \to P(\mathbf{Z})$; $S \mapsto S \cap \mathbf{Z}$.

(k) **Inj N**: $P(\mathbf{Z}) \to P(\mathbf{Z})$; $S \mapsto S \cup \{1\}$.

(l) **Inj N**: $\mathbf{Z} \to P(\mathbf{Z})$; $n \mapsto \{n\}$.

(m) **Inj N**: $P(\mathbf{Z}) \to \mathbf{Z}$; $S \mapsto |S|$ if S is finite, 0 if S is infinite.

(n) **Inj N**: $\mathbf{R} \to \mathbf{R}$; $x \mapsto x^3 + 1$.

(o) **Inj N**: $\mathbf{R} \to \mathbf{R}$; $x \mapsto x(x^2 - 1)$.

(p) **Inj N**: $\mathbf{C} \to \mathbf{C}$; $x \mapsto x^2$.

(q) **Inj N**: $[1, \infty) \to [0, \infty)$; $x \mapsto x^3 - x$.