USEFUL BOOLEAN IDENTITIES

Here are some basic identities.

(1) $P \wedge Q = Q \wedge P$
(2) $P \vee Q = Q \vee P$
(3) $(P \wedge Q) \wedge R = P \wedge (Q \wedge R) = P \wedge Q \wedge R$
(4) $(P \vee Q) \vee R = P \vee (Q \vee R) = P \vee Q \vee R$

Here are some less basic identities.

(1) $\neg(P \wedge Q) = \neg P \vee \neg Q$
(2) $\neg(P \vee Q) = \neg P \wedge \neg Q$
(3) $\neg(\neg P) = P$
(4) $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$
(5) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$
(6) $\neg(P \Rightarrow Q) = P \wedge \neg Q$
(7) $\neg(\forall x, P(x)) = \exists x$ such that $\neg P(x)$
(8) $\neg(\exists x$ such that $P(x)) = \forall x, \neg P(x)$

We can combine these to negate more complicated statements

(1) $\neg(P \Rightarrow (Q \vee R)) =$
$P \wedge \neg(Q \vee R)) =$
$P \wedge \neg Q \wedge \neg R$

(2) If $1 = 0$ or $2 + 2 = 5$, then the sky is blue and kittens are cute.
If (P or Q) then (R and T)

Its negation:
(P or Q) and not (R and T)
($1 = 0$ or $2 + 2 = 5$) and (the sky is not blue or kittens are not cute)

(3) Let's negate $\neg Q \Rightarrow \neg P$.

$\neg(\neg Q \Rightarrow \neg P)$
$\neg Q \wedge \neg(\neg P)$
$\neg Q \wedge P$

This last example is called the contrapositive, and is a useful proof technique! (Try it on your homework.)

(1) $(P \Rightarrow Q) = (\neg Q \Rightarrow \neg P)$ because they have the same negation.