Math 220: Mathematical Reasoning and Proof
Instructor: David Zureick-Brown ("DZB")

# All Solutions

Last updated: September 4, 2026

Solutions 1: Introduction to course. First proof

Solutions to appear after the assignment is due.

1. Using the definitions of ‘divides” and ‘even”, explain why 0 is even. Explain why 7 divides 0, but 0 does not divide 7.

   **Solution.** Recall that an integer $a$ is *even* if there exists an integer $k$ such that $a = 2k$. Since $0 = 2 \cdot 0$ and 0 is an integer, it follows that 0 is even.

   Recall also that, for integers $a$ and $b$, we say that *$a$ divides $b$* if there exists an integer $k$ such that $b = ak$. Since $0 = 7 \cdot 0$, there exists an integer $k$, namely $k = 0$, such that $0 = 7k$. Therefore, $7 \mid 0$.

   On the other hand, if 0 divided 7, then there would be an integer $k$ such that $7 = 0 \cdot k$. But $0 \cdot k = 0$ for every integer $k$, so this would imply that $7 = 0$, which is false. Therefore, $0 \nmid 7$.

2. Suppose that $n$ is an even integer, and let $m$ be any integer. Prove that $nm$ is even.

   **Solution.** Since $n$ is even, there exists an integer $k$ such that $n = 2k$. Multiplying by $m$, we obtain

   $$nm = (2k)m = 2(km).$$

   Because $k$ and $m$ are integers, their product $km$ is an integer. Thus $nm$ is twice an integer, and we conclude that $nm$ is even.

3. Suppose that $n$ and $m$ are odd integers. Prove that $n + m$ is an even integer.

   **Solution.** Since $n$ and $m$ are odd, there exist integers $a$ and $b$ such that $n = 2a + 1$ and $m = 2b + 1$. Therefore,

   $$n + m = (2a + 1) + (2b + 1) = 2a + 2b + 2 = 2(a + b + 1).$$

   Since $a$ and $b$ are integers, $a + b + 1$ is an integer. Thus $n + m$ is twice an integer, and we conclude that $n + m$ is even.