**MATH 250, Foundations of Mathematics**
**Section 005 TuTh 2:30 - 3:45**

**All assignments**
Last updated: November 4, 2021

# Contents

# Week 1

**Topics**: Introduction to the course. Basics of proof. Basics of logic.

**Reading**: Chapter 1, except for proof by contradiction.

**Suggested problems (do not hand in)**

- With answers:
  - Section 1.1, #1(adgj), 2(adji), 3(adgi), 5(ad), 6(a)
  - Section 1.2, #2(ac), 4(ac), 5(ad), 7(a), 10(a), 11(a), 12(a)
  - Section 1.3, #1(ad), 3(a), 5(ac), 7(ac)
  - Section 1.4, #1, 4(a), 6(a), 8, 12(ab), 15(a)
- Handout 1

**Assignment, due Tuesday, August 31, in class**:

1. Suppose that $n$ is an odd integer. Prove that $n^2$ is an odd integer. (Hint: an integer $n$ is odd if and only if there exists an integer $k$ such that $n = 2k + 1$.)
2. Prove that if $n^2$ is even, then $n$ is even. (Hint: see the end of the "useful negation identities" worksheet.)
3. Section 1.1, #4, #5(bcef)

# Week 2

**Topics**: Divisibility problems.

**Reading**:

- Finish reading chapter 1.
- Section 5.3

**Suggested problems (do not hand in)**

1. With answers: Section 5.3, #1(a), 4(a), 6(ac)
2. Without answers: Section 5.3, #2, 4 (without induction), 5 (without induction)
3. Handout 2

**Assignment, due Tuesday, September 7, in class**:

- Section 5.3, #3 (Hint: there are two cases: $x$ is even, $x$ is odd.)
- Handout 2, #6(a), 8, 10

# Week 3

**Topics**: Proof by contradiction. Unsolvability of equations. Irrationality.

**Reading**:

- Section 1.4, p. 41-42 (stop at Historical Comments)
- Section 5.4

**Suggested problems (do not hand in)**

1. Without answers: Section 1.4 #21
2. Without answers: Section 5.4 #6, 7, 10(a), 15, 18,
3. Handout 3

**Assignment, due Tuesday, September 14, in class**:

Section 1.4, #17. (Hint: there are two cases. Either $x$ is even, or $x$ is odd. Consider each case separately and try to get a contradiction.)

Section 5.4, #6(b).

Handout 3, #4, 10.

# Week 4

**Topics**: Induction.

**Reading**: Section 5.2, p. 159-163

**Fun Video**: Vi Hart; "Doodling in Math: Spirals, Fibonacci, and Being a Plant"
https://www.youtube.com/watch?v=ahXIMUkSXX0

**Suggested problems (do not hand in)**

1. With answers: Section 5.2 #1(a), 4(a), 8(ad), 9(a), 29
2. Without answers: Section 5.2 #2-9, 13
3. Handout 4
4. Handout 5

**Assignment, due Tuesday, September 21, in class**:

Section 5.2, #1(bc), #7, 14

Handout 5, #12(f)

# Week 5

**Topics**: Basics of set theory. Basic operations. Proofs with sets.

**Reading**:

1. Section 2.1, p. 49-57;
2. Section 2.2, p. 61-65 (stop at DeMorgan's laws)

**Suggested problems (do not hand in)**

1. With answers (many of these are calculations; do as many as you need to do to understand the definitions):
   (a) Section 2.1, #1(adg), 2(adg), 4(adg), 5(a), 7(a), 8(ae), 9(adf), 10(a), 18(acf), 19(ad), 20(ae), 21
   (b) Section 2.2, #1(adgj), 2(ad), 4(ad), 5(ad), 7(a), 9(ad), 14(a),
2. Without answers:
   (a) Section 2.1, 13, 14, 15, 16,
   (b) Section 2.2, #1-12
3. Handout 6

**No assignment due this week. No office hours on Monday, September 27**.

# Week 6

**Topics**: More proofs with sets. DeMorgan's laws. Cartesian Products. Power sets

**Reading**:

1. Section 2.2, p. 65-66;
2. Section 2.3, p. 72, just the part about power sets.

**Suggested problems (do not hand in)**

1. With answers:
   (a) Section 2.2, 13(a), 16(a)
   (b) Section 2.3, #1(a), 3, 5(adg),
2. Without answers:
   (a) Section 2.2, 14, 16-19, 21, 23-27
   (b) Section 2.3, #1(b), 2,4
3. Handout 7

**Assignment, due Tuesday, October 5, in class**:

Section 2.1, #10(b), 12

Section 2.2, #14(b), #15, 22

Section 2.3, #5 (No proofs necessary)

# Week 7

**Topics**: Exam review. The exam is on Tuesday, October 19.

**Content**: The questions will all be either

1. homework problems,
2. suggested problems,
3. problems we worked in class, or
4. minor variations of one of these.

**No assignment due this week**.

**Fall Break is Monday, October 11 and Tuesday, October 12; no class or office hours these days**.

**Thursday, October 14 will be a review session; please show up with questions**.

# Week 8

**Topics**: Introduction to functions; images and surjectivity

**Reading**:

1. Section 3.1, p. 81-90 (stop at "Inverse Image");
2. Section 3.2, p. 97-100 (stop at Injective Functions).

**Suggested problems (do not hand in)**

1. With answers:

   Section 3.1, #1(adg), 4(ace), 5(a), 8(a), 10(a), 12(1d)

   Section 3.2, #1(adgj), 2(ad),

2. Without answers:

   Section 3.1, 1-4,6-13

   Section 3.2, 1-6

3. Handout 9

**Assignment, due Tuesday, October 26, in class**:

Section 2.3, #6

Section 3.1, #5, 14, 16

# Week 9

**Topics**: Inverse Image (or "Preimage").

**Reading**: Section 3.1, p. 90-92 (stop at the Historical Comments. Or don't.)

**Suggested problems (do not hand in)**

1. With answers: Section 3.1, #17(ad), #18(adg), #19(a), #21(a)
2. Without answers: 17-21
3. Handout 10

**Assignment, due Tuesday, November 2, in class**:

Section 3.1, #18(be) #19(b);

Handout 10, #7, 8.

# Week 10

**Topics**: Injectivity.

**Reading**: Section 3.2, p. 100-105

**Suggested problems (do not hand in)**

1. With answers: 3.2, #12(adg), #13(bd)
2. Without answers: 3.2 #9-14, 19(abc)
3. Handout 11

**Assignment, due Tuesday, November 9, in class**:

Section #3.2, 14(abe) 17, 20, D5

# Week 11

**Topics**: Composition of functions.

**Reading**: Section 3.3, p. 110-113

**Suggested problems (do not hand in)**

1. With answers: 3.3, #1(a), 2(a), 3(ad), 7(a)
2. Without answers: 3.3 #1-7, 9
3. Handout 12

**Assignment, due Tuesday, November 16, in class**:

Section 3.3, #8,

Handout 12, #4(hij), 5(b).

# Week 12

**Topics**: Inverse functions.

**Reading**: Section 3.3, p. 114-116

**Suggested problems (do not hand in)**

1. With answers: 3.3 #10(adgj), 11(a)
2. Without answers: 3.3 #10, 12, 14, 15, 17, 18, 19, 22
3. Handout 13

**Assignment, due Tuesday, November 23, in class**:

Section 3.3, #11, #16, #20, #21

# Week 13

**Topics**: Relations.

**Reading**: Section 4.2, p. 139-144 (stop at the proof of Theorem 4.2.6)

**Suggested problems (do not hand in)**

1. With answers: Section 4.2 #1(a), 3(ad), 4(a), 5(a), 12(a)
2. Without answers: Section 4.2 #1, 3, 4
3. Handout 14

**Assignment, due Tuesday, Dec 7, in class**:

Section 4.2, #2, #5, #10, #18

# Week 14

**Final exam** is Thursday December 9, 3:00pm - 5:30pm

The **last day of class** is December 7. That day will be a review session; please show up with questions.

There will be **office hours** on December 8.